

Observational data fitting to constrain Variable Modified Chaplygin Gas in the background of Horava-Lifshitz Gravity

Chayan Ranjit¹

Department of Mathematics, Egra S. S. B. College, Purba Medinipur-721429, W.B. India.

Prabir Rudra²

Department of Mathematics, Asutosh College, Kolkata-700 026, India.

Abstract

FRW universe in Horava-Lifshitz (HL) gravity model filled with a combination of dark matter and dark energy in the form of variable modified Chaplygin gas (VMCG) is considered. The permitted values of the VMCG parameters are determined by the recent astrophysical and cosmological observational data. Here we present the Hubble parameter in terms of the observable parameters Ω_{dm0} , Ω_{vmcg0} , H_0 , redshift z and other parameters like α , A , γ and n . From Stern data set (12 points), we have obtained the bounds of the arbitrary parameters by minimizing the χ^2 test. The best-fit values of the parameters are obtained by 66%, 90% and 99% confidence levels. Next due to joint analysis with BAO and CMB observations, we have also obtained the bounds of the parameters (A, γ) by fixing some other parameters α and n . The best fit value of distance modulus $\mu(z)$ is obtained for the VMCG model in HL gravity, and it is concluded that our model is perfectly consistent with the union2 sample data.

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1 Introduction

For several decades we have tried to give birth to a well constructed quantum gravity theory that can reconcile with the general theory of Relativity. With the discovery of the late cosmic acceleration [1, 2, 3, 4, 5] at the turn of the last century, the unified theory became all the more necessary. Several footsteps can be found in literature that aims at producing a UV complete theory. Motivated by the success of the theory proposed by Lifshitz [6] in solid state physics, Horava proposed a gravity theory widely known the Horava-lifshitz (HL) gravity [7]. Taking the UV limit into account, HL gravity has a Lifshitz like anisotropic scaling as $t \rightarrow l^z t$ and $x^i \rightarrow l x^i$ between space and time. As this is characterized by the dynamical critical exponent $z = 3$, it breaks the Lorentz invariance, while in the infra red limit, the scale reduces to $z = 1$, i.e., it is reduced to classical general relativistic theory of gravity in the low energy limit. Even if we add a $\frac{1}{a^4}$ term, 'a' being the scale factor, with the Friedmann equation [9, 10, 11, 12, 13, 14, 15] we will have HL gravity equations.

In the original proposal, Horava [7, 8] considered projectability condition with/without the detailed balance. The detailed balance condition was originally proposed in order to reduce the number of operations in the action (i.e., number of independent coupling constants) and to simplify some properties of the quantum system. As a result the form of potential in the 4-dimensional Lorentzian action is restricted to a specific form in terms of a 3D Euclidean theory. But from cosmological point of view, this condition leads to major obstacles and hence should be abandoned.

On the other hand, the fundamental symmetry of the theory namely the foliation-preserving diffeomorphism invariance leads to the projectability condition. In particular, one should have 3D spatial diffeomorphism and

¹chayanranjit@gmail.com

²prudra.math@gmail.com

the space-independent time reparametrization. As the lapse function is essentially the Gauge degree of freedom associated with the time reparametrization, so it should be independent of space coordinates. This is termed as projectibility condition. Due to this projectibility condition, the Hamiltonian constraint is not a local equation satisfied at each spatial point but an equation integrable over a whole space, i.e., a global Hamiltonian constraint. Note that absence of projectibility condition and imposing a local version of the Hamiltonian constraint lead to phenomenological obstacles and theoretical inconsistencies.

However, the important drawback of projectibility condition is that : due to the global Hamiltonian constraint the general relativity could not be recovered from the Horava gravity for arbitrary small λ . Also the projectibility condition from the point of view of condensed matter physics may not be appropriate for describing the (quantum) gravity.

In extragalactic astronomy, type Ia supernovae have a characteristic light curve. The similarity in absolute luminosity profiles of nearly all known type Ia supernovae helps astrophysicists to treat them as a standard candle. Using this fact the observations Riess et al and Perlmutter et al in 1998 [1, 2] led us to the conclusion that our universe is going through a cosmic acceleration. There are two popular ways that provide theoretical support to the present day accelerated expansion of our universe. One is to modify the geometric part of Einstein equation, leading to the ideas of modified gravity and the other is to consider the universe to be filled up uniformly by some exotic matter possessing negative pressure. The Friedmann equation $\frac{\ddot{a}}{a} = -4\pi G(\rho + 3p)/3$ requires the condition $(\rho + 3p) < 0$ for accelerated expansion ($\ddot{a} > 0$). As density is an ever positive physical quantity, we see the Equation of state (EoS) parameter must be negative and also less than $-1/3$. Such a negative pressure creating substance is aptly termed as dark energy (DE here after). Recent observational data indicates that DE occupies 73% of the whole matter-energy of our universe. Theoretically we can find many proposed DE candidates. Variable modified Chaplygin gas (VMCG) is one among them. The equation of state of VMCG is given as [16, 17]

$$p = \alpha\rho - \frac{\beta(a)}{\rho^n} \quad (1)$$

Here we consider $\beta(a) = \beta_0 a^{-\gamma}$, where β_0 and γ are constants.

In 1904, Chaplygin [19] introduced Chaplygin gas (CG), whose EoS is given by

$$p = -\frac{\beta}{\rho} \quad (2)$$

where β is a positive constant. An attractive feature of the model is that it behaves as dust-like matter at an early stage and as cosmological constant at later stages i.e. the CG behaves like a pressure less fluid for small values of scale factor and as a cosmological constant for large values of scale factors which tend to accelerate the expansion. At the beginning of this century, the CG model went through a series of modifications, all in the quest of a suitable candidate for DE.

In [20], Bento et al for the first time gave the idea of Generalised CG having the EoS

$$p = -\frac{\beta}{\rho^n} \quad (3)$$

Whereas in 2010, Jamil [21] presented a model in which the new generalized Chaplygin gas interacts with matter. Benaoum in 2002 further modified CG model and proposed the Modified CG (MCG) model obeying the equation of state [22, 23, 24, 25]

$$p = \alpha\rho - \frac{\beta}{\rho^n} \quad (4)$$

with $\alpha > 0$ and $0 \leq n \leq 1$. When $\alpha = \frac{1}{3}$, this EoS shows radiation era at one extreme (when the scale factor is vanishingly small) while Λ CDM model at the other extreme (when the scale factor is infinitely large). At all stages it shows a mixture. Amidst these there also exist one stage when the pressure vanishes and the matter content is equivalent to pure dust. On further modification VMCG came into existence. In [18], Jamil investigated the evolution of a Schwarzschild black hole in the standard model of cosmology using the phantom-like modified variable Chaplygin gas and the viscous generalized Chaplygin gas.

Present day trend of literature says that the combination of DE with modified gravity together gives more interesting results. In 2010 Park showed in [26] that the Friedmann equation in Horava gravity contains additional a^{-4} , a^{-2} and cosmological constant terms as the effective DE and he predicted that these terms may

be responsible for cosmic acceleration. These terms, coined as Horava effective DE, were tried to be constrained on a few occasions [27, 28]. The best ever approach was via phenomenological parametrization of the relevant physical quantities that have been well-studied. Park himself considered the widely used CPL parametrization. B. C. Paul and his colleagues have studied constraints for different exotic fluid model in the back ground of HL gravity [29, 30]. In this work, we shall study the limits of the DE parameters constrained by different data sets. First we will set the constraint for the closed universe and analyze it. Later on we deal with the range of parameters for open and flat universe. It is quite understood that any observational constraints on HL gravity do not enlighten the discussion about the well-known conceptual problems and instabilities of the theory, nor it can address the questions concerning its validity. Therefore in the present analysis the HL gravity has to be considered as a phenomenological model. The same holds for the Chaplygin Gas model as well.

The paper is organized as follows: The basic generalized equations for HL gravity are given in section (2). Various dimensionless density parameters have been discussed in section (3). The main mechanisms which will be followed to analyze the data is briefly given in section (4). Lastly, a brief summary and a fruitful conclusion have been drawn in section (5).

2 Basic Equations in Horava-Lifshitz Gravity

It is convenient to use the Arnowitt-Deser-Misner decomposition of the metric which is given by [11, 12, 13]

$$ds^2 = -N^2 dt^2 + g_{ij} (dx^i + N^i dt) (dx^j + N^j dt). \quad (5)$$

Here, N is the lapse function, N_i is the shift vector, g_{ij} is the spatial metric. The scaling transformation of the coordinates reads as : $t \rightarrow l^3 t$ and $x^i \rightarrow l x^i$. The HL gravity action has two constituents, namely, the kinetic and the potential term as

$$S_g = S_k + S_v = \int dt d^3 x \sqrt{g} N (L_k + L_v) \quad (6)$$

where, the kinetic term is given by

$$S_k = \int dt d^3 x \sqrt{g} N \left[\frac{2 (K_{ij} K^{ij} - \lambda K^2)}{\kappa^2} \right] \quad (7)$$

where, the extrinsic curvature is given as

$$K_{ij} = \frac{\dot{g}_{ij} - \Delta_i N_j - \Delta_j N_i}{2N} \quad (8)$$

The number of invariants, while working with the Lagrangian, L_v , can be reduced due to its symmetric property [8]. This symmetry actually is known as detailed balance. Considering this detailed balance the expanded form of the action becomes

$$S_g = \int dt d^3 x \sqrt{g} N \left[\frac{2 (K_{ij} K^{ij} - \lambda K^2)}{\kappa^2} + \frac{\kappa^2 C_{ij} C^{ij}}{2\omega^4} - \frac{\kappa^2 \mu \epsilon^{ijk} R_{i,j} \Delta_j R_k^l}{2\omega^2 \sqrt{g}} \right. \\ \left. + \frac{\kappa^2 \mu^2 R_{ij} R^{ij}}{8} - \frac{\kappa^2 \mu^2}{8(3\lambda - 1)} \left\{ \frac{(1 - 4\lambda) R^2}{4} + \Lambda R - 3\Lambda^2 \right\} \right] \quad (9)$$

here $C^{ij} = \frac{\epsilon^{ijk} \Delta_k (R_i^j - \frac{R}{4} \delta_i^j)}{\sqrt{g}}$ is the Cotton tensor and all the covariant derivatives are determined with respect to the spatial metric. $g_{ij} \epsilon^{ijk}$ is a totally antisymmetric unit tensor, λ is a dimensionless constant and κ , ω and μ are constants.

Now to set the matter-tensor the total energy density and pressure, ρ_m and p_m respectively will be taken which will contain in itself the impact of the baryonic density (ρ_b), dark matter density (ρ_{dm}), etc.

Assuming only temporal dependency of the lapse function (i.e., $N \equiv N(t)$), Horava obtained a gravitational action. Using FRW metric with $N = 1$, $g_{ij} = a^2(t) \gamma_{ij}$, $N^i = 0$ and

$$\gamma_{ij} dx^i dx^j = \frac{dr^2}{1 - Kr^2} + r^2 d\Omega_2^2$$

where $K = -1, 1, 0$ represent open, closed and flat universe respectively and taking variation of N and g_{ij} we obtain the Friedmann equations [14, 30]

$$H^2 = \frac{\kappa^2}{6(3\lambda-1)}(\rho_m + \rho_r) + \frac{\kappa^2}{6(3\lambda-1)} \left[\frac{3\kappa^2\mu^2 K^2}{8(3\lambda-1)a^4} + \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)} \right] - \frac{\kappa^4\mu^2\Lambda K}{8(3\lambda-1)^2 a^2} \quad (10)$$

$$\dot{H} + \frac{3H^2}{2} = -\frac{\kappa^2}{4(3\lambda-1)}(\rho_m w_m + \rho_r w_r) - \frac{\kappa^2}{4(3\lambda-1)} \left[\frac{3\kappa^2\mu^2 K^2}{8(3\lambda-1)a^4} + \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)} \right] - \frac{\kappa^4\mu^2\Lambda K}{8(3\lambda-1)^2 a^2} \quad (11)$$

The term proportional to $\frac{1}{a^4}$ is an unique contribution of HL gravity, which can be treated as “Dark radiation term” [11, 12, 13] and the constant term is the cosmological constant.

The conservation equation of matter is

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0 \quad (12)$$

and that of radiation is

$$\dot{\rho}_r + 3H(\rho_r + p_r) = 0 \quad (13)$$

where, $G_{cosmo} = \frac{\kappa^2}{16\pi(3\lambda-1)}$ with the conditions $\frac{\kappa^4\mu^2\Lambda}{8(3\lambda-1)} = 1$ and $G_{grav} = \frac{\kappa^2}{32\pi}$.

Solving the DE conservation equation for variable modified Chaplygin Gas, the expression for DE density for VMCG will be

$$\rho_{vmcg} = \rho_{vmcg0}(1+z)^{3(1+n)} [A + (1-A)(1+z)^\gamma]^{\frac{1}{\alpha+1}}. \quad (14)$$

where $\rho_{vmcg0} = \left[-\frac{3(n+1)\beta_0}{\gamma} + C \right]^{\frac{1}{\alpha+1}}$, $A = \frac{C}{\rho_{vmcg0}^{\alpha+1}}$, where C is the integration constant.

3 Observational Constraints on EOS Parameters

Using Eqs. (10)-(11), the Friedmann's equation can be rewritten as:

$$H^2 = \frac{8\pi G}{3}(\rho_{vmcg} + \rho_{dm} + \rho_r) + \left(\frac{K^2}{2\Lambda a^4} + \frac{\Lambda}{2} \right) - \frac{K}{a^2} \quad (15)$$

$$\dot{H} + \frac{3H^2}{2} = -4\pi G \left(p_{vmcg} + \frac{1}{3}\rho_r \right) - \left(\frac{K^2}{4\Lambda a^4} - \frac{3\Lambda}{4} \right) - \frac{K}{2a^2} \quad (16)$$

We define the following dimensionless density parameters:

(i) for matter component:

$$\Omega_{i0} = \frac{8\pi G}{3H_0^2} \rho_{i0} \quad (17)$$

(ii) for curvature:

$$\Omega_{K0} = -\frac{K}{H_0^2} \quad (18)$$

(iii) for cosmological constant:

$$\Omega_0 = \frac{\Lambda}{2H_0^2} \quad (19)$$

Another dimensionless parameter for expansion rate is defined as:

$$E(z) = \frac{H(z)}{H_0} \quad (20)$$

Using the above parameters, the Friedmann equation can be rewritten as:

$$E^2(z) = \Omega_{vmcg0}F(z) + \Omega_{dm0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \frac{\Omega_{K0}^2}{4\Omega_0}(1+z)^4 + \Omega_0 + \Omega_{K0}(1+z)^2 \quad (21)$$

where

$$F(z) = \rho_{vmcg0}(1+z)^{3(1+n)} [A + (1-A)(1+z)^\gamma]^{\frac{1}{\alpha+1}} \quad (22)$$

Here Ω_{r0} is for radiation and Ω_{dm0} is for dark matter. At the present epoch $E(z=0) = 1$, which leads to

$$\Omega_{vmcg0} + \Omega_{dm0} + \Omega_{r0} + \frac{\Omega_{K0}^2}{\Omega_0} + \Omega_0 + \Omega_{K0} = 1 \quad (23)$$

4 Observational Data Analysis In HL Universe

In this section, we perform a detailed observational data analysis [31, 32, 33, 34, 35] using Stern data. We also study the model under Stern+BAO and Stern+BAO+CMB joint analysis. The mechanism that we will use in the present work is the χ^2 minimum test from theoretical Hubble parameter with the observed data set and find the best fit values of unknown parameters for different confidence levels (66%, 90%, 99%). In the table given below, we present the 3 column Stern data.

z	$H(z)$	$\sigma(z)$
0	73	± 8
0.1	69	± 12
0.17	83	± 8
0.27	77	± 14
0.4	95	± 17.4
0.48	90	± 60
0.88	97	± 40.4
0.9	117	± 23
1.3	168	± 17.4
1.43	177	± 18.2
1.53	140	± 14
1.75	202	± 40.4

Table 1: The Hubble parameter $H(z)$ and the standard error $\sigma(z)$ for different values of redshift z .

4.1 Stern ($H(z)$ - z) Data Set

In this sub-section our theoretical model of VMCG in HL gravity is analyzed, using the observed values of Hubble parameter at different redshifts (twelve data points) listed in observed Hubble data by Stern et al [36]. The observed values of Hubble parameter $H(z)$ and the standard error $\sigma(z)$ for different values of redshift z are given in Table 1. A statistical hypothesis is proposed and its validity is tested at different confidence levels. For this purpose we first form the χ^2 statistics as a sum of standard normal distribution as follows:

$$\chi_{Stern}^2 = \sum \frac{(H(z) - H_{obs}(z))^2}{\sigma^2(z)} \quad (24)$$

$$L = \int e^{-\frac{1}{2}\chi_{Stern}^2} P(H_0) dH_0 \quad (25)$$

where $H(z)$ and $H_{obs}(z)$ are theoretical and observational values of Hubble parameter at different redshifts respectively. Here, H_{obs} is a nuisance parameter and can be safely marginalized. H_0 is the present value of Hubble parameter and its value is fixed at $H_0 = 72 \pm 8 \text{ Kms}^{-1} \text{ Mpc}^{-1}$. From the DE model we see that the two most important parameters are A and $B(a)$. Here we shall determine best fit value of the parameters (A vs γ) by minimizing the above distribution χ_{Stern}^2 and fixing the other unknown parameters with the help of Stern data. According to our analysis the best fit values of A vs γ is presented in Table 2. We now plot the graph for different confidence levels. Our best fit analysis with Stern observational data support the theoretical range of the parameters. The 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours are plotted in fig.1 for $\alpha = 0.001$, $n = 0.1$, $\Omega_{i0} = 0.01$, $\Omega_{r0} = 0.02$, $\Omega_{K0} = 0.01$, $\Omega_0 = 0.01$, $\Omega_{dm0} = .03$.

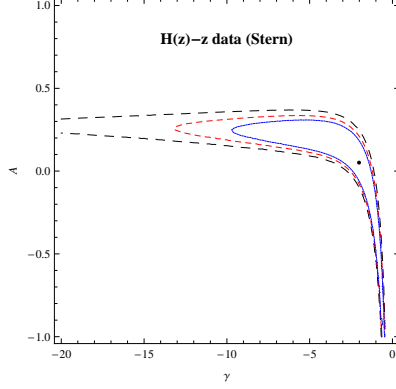


Fig.1

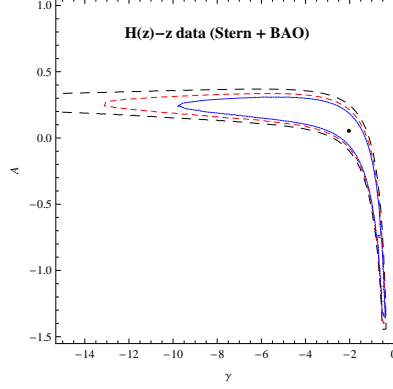


Fig.2

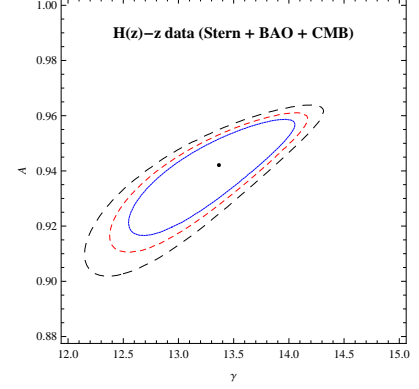


Fig.3

The contours are drawn for 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) confidence levels.

Fig.1 shows the variations of A against γ for $\alpha = 0.001, n = 0.1, \Omega_{i0} = 0.01, \Omega_{r0} = 0.02, \Omega_{K0} = 0.01, \Omega_0 = 0.01, \Omega_{dm0} = .03$ with respectively for the $H(z)$ - z joint analysis.

Fig.2 shows the variations of A against γ for $\alpha = 0.001, n = 0.1, \Omega_{i0} = 0.01, \Omega_{r0} = 0.02, \Omega_{K0} = 0.01, \Omega_0 = 0.01, \Omega_{dm0} = .03$ with respectively for the $H(z)$ - z +BAO joint analysis.

Fig.3 shows the variations of A against γ for $\alpha = 0.001, n = 0.1, \Omega_{i0} = 0.01, \Omega_{r0} = 0.02, \Omega_{K0} = 0.01, \Omega_0 = 0.01, \Omega_{dm0} = .03$ with respectively for the $H(z)$ - z +BAO+CMB joint analysis.

4.2 Stern + BAO Data Sets

Here we resort to a joint analysis in the sense that the BAO peaks are incorporated in the stern data. The Baryon Acoustic Oscillation (BAO) peak parameter value was proposed by [37] and we shall use their approach. The pioneer as far as the detection of BAO signal is concerned, is considered to be the Sloan Digital Sky Survey (SDSS) survey. The survey directly detected the BAO signals at a scale ~ 100 Mpc. The analysis that is followed is actually the combination of angular diameter distance and Hubble parameter at that redshift. This analysis is independent of the measurement of H_0 and does not contain any particular dark energy. Here we examine the parameters A vs γ for VMCG model from the measurements of the BAO peak for low redshift (with range $0 < z < 0.35$) using standard χ^2 analysis. The error corresponds to the standard deviation, where the distribution considered is Gaussian. We know that the Low-redshift distance measurements are very lightly dependent on different cosmological parameters, the EoS of dark energy and have the ability to measure the Hubble constant H_0 directly. The BAO peak parameter is defined by

$$\mathcal{A} = \frac{\sqrt{\Omega_m}}{E(z_1)^{1/3}} \left(\frac{1}{z_1} \int_0^{z_1} \frac{dz}{E(z)} \right)^{2/3} \quad (26)$$

Here $E(z) = H(z)/H_0$ is the normalized Hubble parameter, the redshift $z_1 = 0.35$ is the typical redshift of the SDSS sample and the integration term is the dimensionless comoving distance for the redshift z_1 . The value of the parameter \mathcal{A} for the flat model of the universe is given by $\mathcal{A} = 0.469 \pm 0.017$ using SDSS data [37] from luminous red galaxies survey. Now the χ^2 function for the BAO measurement is given as,

$$\chi_{BAO}^2 = \frac{(\mathcal{A} - 0.469)^2}{(0.017)^2} \quad (27)$$

The total joint data analysis (Stern+BAO) for the χ^2 function may be defined by

$$\chi_{total}^2 = \chi_{Stern}^2 + \chi_{BAO}^2 \quad (28)$$

According to our analysis the best fit values of A vs γ for the joint scheme is presented in Table 2. Finally we generate the closed contours of A vs γ for the 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) confidence limits depicted in fig.2 for $\alpha = 0.001, n = 0.1, \Omega_{i0} = 0.01, \Omega_{r0} = 0.02, \Omega_{K0} = 0.01, \Omega_0 = 0.01, \Omega_{dm0} = .03$.

4.3 Stern + BAO + CMB Data Sets

The angular scale of the first acoustic peak is measured through angular scale of the sound horizon at the surface of last scattering. This is one of the most interesting geometrical probe of dark energy. The information is encoded in the CMB (Cosmic Microwave Background) power spectrum. The definition of the CMB shift parameter is given in [38, 39, 40]. It is not sensitive with respect to perturbations but are suitable to constrain model parameter. This is the property that we will use in this analysis. The CMB power spectrum first peak is the shift parameter which is given by

$$\mathcal{R} = \sqrt{\Omega_m} \int_0^{z_2} \frac{dz}{E(z)} \quad (29)$$

where z_2 is the value of redshift at the last scattering surface. From WMAP7 data of the work of Komatsu et al [41] the value of the parameter was obtained as $\mathcal{R} = 1.726 \pm 0.018$ at the redshift $z = 1091.3$. The χ^2 function for the CMB measurement can be written as

$$\chi_{CMB}^2 = \frac{(\mathcal{R} - 1.726)^2}{(0.018)^2} \quad (30)$$

Now when we consider three cosmological tests together, the total joint data analysis (Stern+BAO+CMB) for the χ^2 function is defined by

$$\chi_{TOTAL}^2 = \chi_{Stern}^2 + \chi_{BAO}^2 + \chi_{CMB}^2 \quad (31)$$

Now the best fit values of (A, γ) for joint analysis of BAO and CMB with Stern observational data support the theoretical range of the parameters are given in Table 2. The 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours are plotted in fig.3 for $\alpha = 0.001, n = 0.1, \Omega_{i0} = 0.01, \Omega_{r0} = 0.02, \Omega_{K0} = 0.01, \Omega_0 = 0.01, \Omega_{dm0} = .03$.

<i>Data</i>	<i>A</i>	<i>γ</i>	χ_{min}^2
<i>Stern</i>	0.0512871	-2.02163	7.17375
<i>Stern + BAO</i>	0.0536315	-2.03587	768.149
<i>Stern + BAO + CMB</i>	0.942108	13.3671	9554.2

Table 2: The best fit values of A , γ and the minimum values of χ^2 .

4.4 Redshift-Magnitude Observations from Supernovae Type Ia

The main evidence for the existence of dark energy was provided by Supernova Type Ia experiments. The existence of dark energy is directly related to the redshift of the universe. Therefore, since 1995 two teams of High redshift Supernova Search and the Supernova Cosmology Project have been working extensively, and in their effort they have discovered several type Ia supernovae at the high redshifts [1, 2, 3, 42]. The observations directly measure the distance modulus of a Supernovae and its redshift z [2, 43]. Here we will consider the recent observational data, including SNe Ia which consists of 557 data points and belongs to the Union2 sample [44]. From the observations, the luminosity distance $d_L(z)$ will determine the dark energy density which is defined by

$$d_L(z) = (1 + z)H_0 \int_0^z \frac{dz'}{H(z')} \quad (32)$$

The distance modulus (distance between absolute and apparent luminosity of a distance object) for Supernovae is given by

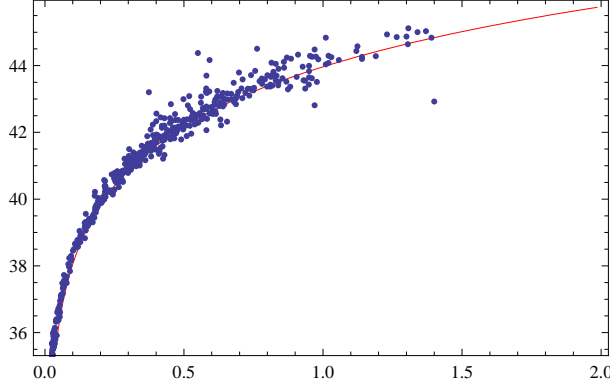


Fig.4: $\mu(z)$ vs z for $n = 0.1, \alpha = 0.01$

$$\mu(z) = 5 \log_{10} \left[\frac{d_L(z)/H_0}{1 \text{ Mpc}} \right] + 25 \quad (33)$$

The best fit of distance modulus as a function $\mu(z)$ of redshift z for our theoretical model and the Supernova Type Ia Union2 sample are drawn in fig.4 for our best fit values of the parameters. From the curve, we see that the theoretical VMCG model in HL gravity is in agreement with the union2 sample data.

5 Discussions

In this work, we have considered the FRW universe in HL gravity model filled with a combination of dark matter and dark energy in the form of variable modified Chaplygin gas (VMCG). We present the Hubble parameter in terms of the observable parameters Ω_{i0} , Ω_{K0} , Ω_0 , Ω_{dm0} , Ω_{vmcg0} and H_0 with the redshift z and the other parameters like A , n , γ and α . For these parameters we have chosen the specific numerical values consistent with observations. From Stern data set (12 points), we have obtained the bounds of the arbitrary parameters by minimizing the χ^2 test. Next due to joint analysis of BAO and CMB observations, we have also obtained the best fit values and the bounds of the parameters (A, γ) . We have plotted the statistical confidence contour of (A, γ) for different confidence levels i.e., 66%(dotted, blue), 90%(dashed, red) and 99%(dashed, black) confidence levels by fixing observable parameters Ω_{dm0} , Ω_{r0} , Ω_{K0} , Ω_0 and H_0 and some other parameters like, n and α , etc. for Stern, Stern+BAO and Stern+BAO+CMB data analysis.

From the Stern data, the best-fit values and bounds of the parameters (A, γ) are obtained. The output values are shown in Table 1 and the figure 1 shows statistical confidence contour for 66%, 90% and 99% confidence levels. Next due to joint analysis with Stern + BAO data, we have also obtained the best-fit values and bounds of the parameters (A, γ) . The results are displayed in the second row of Table 1 and in figure 2 we have plotted the statistical confidence contour for 66%, 90% and 99% confidence levels. After that, due to joint analysis with Stern+BAO+CMB data, the best-fit values and bounds of the parameters (A, γ) are found and the results are shown in Table 1. The figure 3 shows statistical confidence contour for 66%, 90% and 99% confidence levels. For each case, we compare the model parameters through their values and by the statistical contours. From the comparative study, one can get an idea about the convergence of theoretical values of the parameters with their values obtained from the observational data set and how it is altered for different chosen set of other parametric values.

Finally the distance modulus $\mu(z)$ against redshift z has been drawn in figure 4 for the theoretical model of the VMCG in HL gravity for the best fit values of the parameters and the observed SNe Ia Union2 data sample. So the observational data sets are perfectly consistent with our predicted theoretical VMCG model in HL gravity.

The present study discover the constraint of allowed composition of matter-energy by constraining the range of the values of the parameters for a physically viable VMCG in HL gravity model. In a nut-shell, the conclusion of this discussion suggests that even though the quantum aspect of gravity have small effect on the observational constraint, but the cosmological observation can put upper bounds on the magnitude of the correction coming

from quantum gravity that may be closer to the theoretical expectation than what one would expect.

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